RATIO TEST

IF $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$, THEN $\sum_{n=1}^{\infty} a_n$ CONV

IF $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| > 1$ OR $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty$, THEN $\sum_{n=1}^{\infty} a_n$ DIV

IF $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$, OR DNE THEN NO CONCLUSION IS POSSIBLE WITHOUT ADDITIONAL ANALYSIS

eg. CONSIDER $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{e^n}$

$$r = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \boxed{\lim_{n\to\infty}\left|\frac{\frac{(-1)^{n+1}(n+1)!}{e^{n+1}}}{\frac{(-1)^n n!}{e^n}}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}(n+1)!}{e^{n+1}}\,\frac{e^n}{(-1)^n n!}\right|}$$

$$= \lim_{n\to\infty}\left|\frac{(-1)^{n+1}}{(-1)^n}\,\frac{(n+1)!}{n!}\,\frac{e^n}{e^{n+1}}\right| = \boxed{\lim_{n\to\infty}\left|\frac{\cancel{(-1)}(n+1)}{e}\right| = \lim_{n\to\infty}\frac{n+1}{e}} = \boxed{\infty}$$

$$\frac{(n+1)\cancel{n!}}{\cancel{n!}}$$

SO $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{e^n}$ $\boxed{\text{DIV (RATIO TEST)}}$ $\boxed{r=\infty}$

CONSIDER $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(7n+2)}{n^2+5}$

<u>RATIO TEST</u>

$$r = \lim_{n\to\infty} \left| \frac{\dfrac{\cancel{(-1)^{n+1}}(7(n+1)+2)}{(n+1)^2+5}}{\dfrac{\cancel{(-1)^n}(7n+2)}{n^2+5}} \right| = \lim_{n\to\infty} \frac{7n+9}{7n+2} \cdot \frac{n^2+5}{(n+1)^2+5}$$

$$\frac{n^2+5}{(n+1)^2+5} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$= \frac{1+\frac{5}{n^2}}{\frac{(n+1)^2}{n^2}+\frac{5}{n^2}}$$

$$= \frac{1+\frac{5}{n^2}}{\left(\frac{n+1}{n}\right)^2+\frac{5}{n^2}}$$

$$= \lim_{n\to\infty} \frac{7+\frac{9}{n}}{7+\frac{2}{n}} \cdot \frac{1+\frac{5}{n^2}}{\left(1+\frac{1}{n}\right)^2+\frac{5}{n^2}}$$

$$= \frac{7+0}{7+0} \cdot \frac{1+0}{(1+0)^2+0}$$

$$= 1$$

NO CONCLUSION —

NEED TO RUN ANOTHER TEST
TO DETERMINE CONV/DIV

CONSIDER $\displaystyle\sum_{n=1}^{\infty} \frac{(2n+3)^5 e^{4n}}{n!}$

$$r = \boxed{\lim_{n\to\infty} \left| \frac{(2(n+1)+3)^5 e^{4(n+1)}}{(n+1)!} \cdot \frac{n!}{(2n+3)^5 e^{4n}} \right|}$$

$$e^{4(n+1)-4n}$$

$$= \lim_{n\to\infty} \frac{(2(n+1)+3)^5}{(2n+3)^5} \cdot \frac{e^{4(n+1)}}{e^{4n}} \cdot \frac{n!}{(n+1)!}$$

$$= \lim_{n\to\infty} \left(\frac{2(n+1)+3}{2n+3}\right)^5 \cdot e^4 \cdot \frac{1}{n+1}$$

$$= \boxed{\lim_{n\to\infty} \underbrace{\left(\frac{2(1+\frac{1}{n})+\frac{3}{n}}{2+\frac{3}{n}}\right)^5}_{} \cdot \frac{e^4}{n+1}}$$

$$= 1^5 \cdot \lim_{n\to\infty} \cancel{1^5} \cdot \frac{e^4}{n+1}$$

$$\boxed{= 0} \boxed{< 1}$$

$$\boxed{\sum_{n=1}^{\infty} \frac{(2n+3)^5 e^{4n}}{n!} \text{ CONV (RATIO TEST)}}$$

CONSIDER $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

$$\boxed{r = \lim_{n\to\infty} \left| \frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!} \right|}$$

$$= \lim_{n\to\infty} \frac{(n+1)!}{n!} \cdot \frac{n^n}{(n+1)^{n+1}}$$

$$= \lim_{n\to\infty} \cancel{(n+1)} \cdot \frac{n^n}{(n+1)^n \cancel{(n+1)}}$$

$$= \boxed{\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n}$$

$\frac{n}{n+1} \to 1$

1^∞ <u>INDET</u> $\to$ L'H

$$= \cancel{\lim_{n\to\infty} (}$$

$$\boxed{= e^{-1}} \boxed{< 1}$$

$$\boxed{\text{SO } \sum_{n=1}^{\infty} \frac{n!}{n^n} \text{ CONV (RATIO)}}$$

$$\ln\left(\frac{n}{n+1}\right)^n$$

$$= n \ln\frac{n}{n+1}$$

$$= n\left(\ln n - \ln(n+1)\right)$$

$\infty(\underbrace{\infty-\infty})$ $\to$ INDET

~~$\infty \cdot 0 \to$ INDET~~

$$\lim_{n\to\infty} \frac{\ln n - \ln(n+1)}{\frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{n} - \frac{1}{n+1}}{-\frac{1}{n^2}} \cdot \frac{-n^2}{-n^2}$$

$$= \lim_{n\to\infty} \left(-n + \frac{n^2}{n+1}\right)$$

$-\infty + \infty \longrightarrow$ INDET

$$= \lim_{n\to\infty} \frac{\cancel{-n^2} - n + \cancel{n^2}}{n+1}$$

$$= \lim_{n\to\infty} \frac{-n}{n+1}$$

$$= \lim_{n\to\infty} \frac{-1}{1+\frac{1}{n}} = -1$$